



# ICE HANDBOOK FOR ENGINEERS

VERSION 1.1



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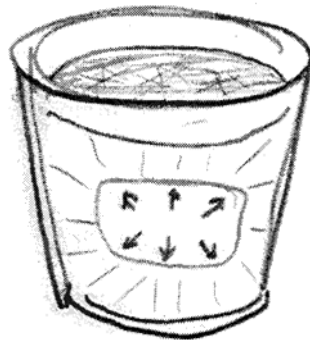


## 1. Ice properties

### ***Frozen water***

In our Nordic climate a water bucket is likely freeze to ice<sup>1</sup> if we leave it outdoors wintertime. The ice is first observed as a thin shell around the water volume and then more ice develops radial towards the centre. In the microscope you can observe the boarder between water and ice, it looks like a zigzag pattern. The freezing front is irregular due to the turbulent heat flow at this borderline of phase change. When water freezes small gas bubbles are released and shrinks due to cooling. Therefore the freezing direction can be backtracked as bubble bands or tubes going from the free water surface to the first frozen ice shell. The degree of overpressure, water movement, temperature gradient and absolute temperature affects the final quality, the ice properties.

Latent heat (334 kJ/kg) is released at the moment of phase change and transported in the direction of colder spots out to the cold air. The heat flow is proportional to the temperature difference of two nearby points. Therefore ice grows quickly in the beginning when it is thin but the freezing rate slows down when the ice becomes thick. Ice can be considered as an insulating material even though it is a poor insulation. Phase change of water is accompanied with volume increase (9 %) that causes high water pressure in the trapped water in the bucket. This pressure may be big enough to break the confinement letting the water out. The type of ice that is created during confined conditions appears to be whitish and soft. In some cases the bucket will be damaged due to the strong overpressure. If insulation is put on top of the bucket the surface will blast off at small pressures and the ice becomes clear.



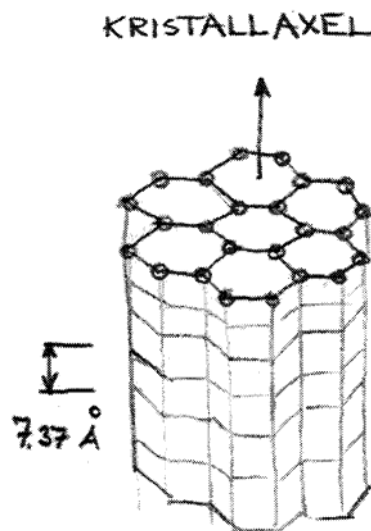
Figur 1 Internal pressure when a bucket of water is multidirectional frozen.

<sup>1</sup> ICE-1 is the only type of ice that exists naturally on earth. ICE-1 is often referred to as Ih due to the hexagonal crystal structure. Chemical clean water do not freeze before  $-40^{\circ}\text{C}$  but natural inclusions in water means that water usually freezes at  $0^{\circ}\text{C}$  and the triple point is  $-0.01^{\circ}\text{C}$ . At high pressures and low temperatures nine more artificial ice types have been observed ICE-2, ICE-3, ... ICE-10. These ice types have different crystal structure and density than ICE-1.



Ice consists of the same basic elements as ordinary water  $H_2O$ , i.e. hydrogen and oxygen. Natural ice consists of many ice crystals, from less than 1 mm to metres. The geometrical structure of each single crystal has a hexagonal symmetry where oxygen atoms and hydrogen bonds form hexagons. These so called base planes pile up with a constant distance of 0.737 nano-meter ( $1 \text{ nm} = 10^{-9} \text{ m}$ ). The main symmetry axis is perpendicular to the base planes and it is referred to as the crystal axis. In a single crystal the crystal axis is equal with the optical axis where light passes without refraction. The crystal axis can easily be determined for ice by turning the crystal until it appears black in cross-polarised light. A thin section with many crystals or grains show different colours depending on crystal axis orientation relatively to the viewing direction.

By measuring the distance between mass centres of oxygen atoms it is possible to build an atomic model of the ice structure. From this geometrical information a theoretical density of ice Ih can be calculated. At a temperature of  $0^\circ\text{C}$  the theoretical density becomes  $916.6 \text{ kg/m}^3$  and it is close to results from experiments with weighing a well-sized piece of pure ice. The single crystal has different properties in different symmetry axis. The mechanical properties of a single crystal can be described as a deck of cards with bad glue between the cards. It is simple to shear the deck but it can take high compressive load. For ice with many crystals (polycrystalline ice) the mechanical behaviour can be deduced from single crystal orientation but also from the shape and size of the crystals and the crystal boundaries.



Figur 2. Crystal axis and basal planes of a single crystal of ice Ih.

### Continuum behaviour

Ice and heated steel show similarities in their deformation behaviour. Maybe this is because they are both polycrystalline materials close to their respectively melting points. If a load is applied under a long time ice can deform substantially without visible cracks. Warm fine



crystal ice (glaciers) will easily deform but cold large crystal lake ice is more brittle. Therefore temperature is one important parameter to know when trying to estimate the actual mechanical properties of a certain ice type.

At high loading rates and small stress levels it is possible to approximate ice behaviour with a linear elastic material model. This means that deformation is assumed proportional to the applied load and that no remaining deformation is seen after the load has been removed. Hook's law applied on ice results in an *elastic modulus* of 5 – 9 GPa and a tensile strength of about 1 MPa. These numbers shows that fracture occurs after only 0.02 % relative elongation, strain. As for many brittle materials the compressive strength is higher than the tensile strength. By introducing fracture mechanics it is possible to show that the expected compressive strength of ice is the tensile strength divided with *Poisson's ratio* 0.1-0.3, which yields a compressive strength of 3 -10 MPa.

At slow loading rates creep plays an important role for the deformation. Creep means that the deformation increases with time when the load is constant. This time-dependent deformation of ice increases exponential with the stress level. Again temperature is an important input and close to the melting point ice grains starts to glide apart. Extremely small inclusions of salt change the creep behaviour of ice close to the melting point.

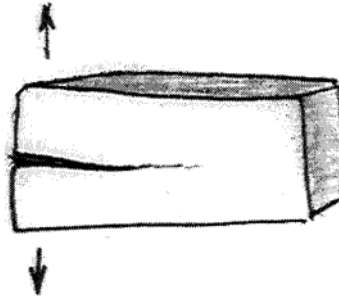
Like most crystalline materials ice expands when it is heated. In the laboratory the thermal expansion can be measured as the relationship between strain  $\varepsilon = \Delta L/L_0$  and the temperature increase  $\Delta T$ . In a small temperature interval the thermal strain is  $\varepsilon(\Delta T) = \alpha \Delta T$ , where  $\alpha$  is referred to as the thermal expansion coefficient. Freshwater ice has its maximum length when it is formed, at zero degrees. A typical value for freshwater ice is  $\alpha = 0.00005 \text{ C}^{-1}$ . The total expansion of a 100 m long ice block that heated from  $-10 \text{ C}$  to 0 will then be  $\Delta L = 0.05 \text{ m}$ .

### Fracture mechanical behaviour

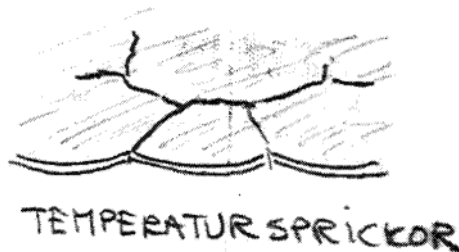
Cold ice is a brittle material and the strength depends on built-in defects in the crystal structure and small cracks. *Fracture toughness* is one parameter that reflects the sensitivity for tensile stress around a crack tip. Tests on small samples have resulted in a value on the fracture toughness of about  $100 \text{ kPa m}^{0.5}$ . It is however still questioned whether ice shows size effects, i.e. if the ice volume or the length of the crack is important when determining the fracture toughness. A natural ice cover on a lake contains thousands of large dry cracks within 10 by 10 metres, formed due to temperature shrinkage on the surface. These hairfine cracks are noticed as a time delay if thermal expansion and internal pressure is measured of a confined ice sheet. Also the bending strength is reduced when the ice surface is put into tension. Other types of thermal bending cracks are more visible. The ice bottom is always around zero and the resulting bending moment when the surface is shrinking causes honey-comb shaped crack pattern clearly seen on the top surface. This type of cracking is accompanied with loud sounds and people say that the ice moos (swe."isen råmar"). When the temperature rises the ice expands and if new ice has refrozen in the cracks the ice cover becomes too large. Ridges may then form along the thermal crack lines when the ice cover expands as when the temperature rises. If wind or currents drives the ice further against each



other ice will gather at the ridge. This phenomenon called pressure ridging is more common at the open sea than at a lake.



Figur 3 Fracturing of an ice block by applying a tensile stress at an existing crack tip.



Figur 4 Thermal cracks on a shrinking ice sheet.

## Lake ice

### Crystal structure

Ice formation on a lake in a calm and cold night takes place in different stages: first there is a network of long floating needles, thereafter the meshes are filled with extremely thin and transparent ice shells. These shells comprise only a few large grains with random orientations of the crystal axes. The ice crystal grows faster along the crystal axis which means that vertical crystals steal space from other grains and finally dominates the ice cover when it is 5 cm and thicker. Horizontal thin sections appear to be black and white in crossed-polarised light. This ice type is referred to as *columnar ice* because the crystal structure is columnar like vertical fingers close-packed and bonded to each other. The columns become gradually larger in diameter as the depth increases. This ice is transparent with a blue shade but sometimes the word black ice is used because it appears as black on deep water. It can however be misleading because black ice is the term used for rotten spring ice in many countries.

The rate of ice growth depends mainly on the cooling from the air, the heat flow. This is in an engineering sense proportional to the heat conductivity upwards from the freezing front. Using one-dimensional and quasi-stationary heat flow equations shows that the relationship between freezing index after ice formation date  $S$  and the ice thickness  $h$  has the form

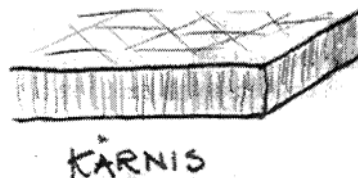


$$h^2 = k_0 S \quad (1)$$

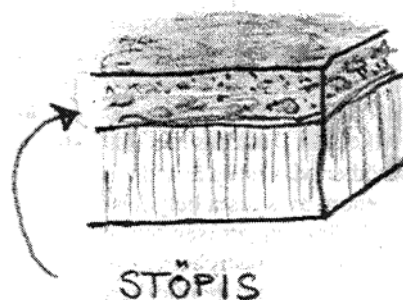
If we are using the units °C, days, cm,  $k_0 = 4.8$  gives reasonable values on  $h$ [cm] in the Lule River as long as it is not too much snow during the winter (Fransson, 2002).  $k_0$  is a lumped constant reflecting the conductivity, density and latent heat of formed ice. Snow tends to decrease the value on  $k_0$  due to an extra insulating effect. This effect is however compensated by the fact that flooded snow creates granular snow-ice on top of the columnar freshwater ice. Many parameters other than freezing index influence the freezing rate of columnar ice. When the ice is thin evaporation and temperature of surface water is important in the heat budget. A useful empirical formula first proposed by Zubov (1938) is

$$h^2 + k_1 h = k_2 S \quad (2)$$

where  $k_1$  and  $k_2$  are empirical constants. If it is windy or if it is a snowfall at the time of primary ice formation the ice surface becomes rough and uneven. The thin ice shells breaks and snow mixes with water and ice into an undefined structure. This affects the ice growth process and the size of the grains becomes small and randomly oriented. Secondary underlying ice is still transparent and columnar but due to smaller grains it is more ductile than the ice that was formed during calm weather conditions. If more snow falls down on the ice it will be submerged and flooded. The flooded water mixes with the snow and the mixture freezes after some cold period. This milk-coloured ice type is called *snow-ice*. The properties of snow-ice vary with air bubble content which can be estimated from average density. The size of the air bubble and the geometrical pattern are however not shown if only density is measured. When snow-ice freezes together with columnar ice it gives an added bearing capacity, but as long as unfrozen water can be observed between the two layers the effect is only negative and ice growth under the ice cover is effectively stopped.



Figur 5 Columnar ice on a lake, transparent and bluish.



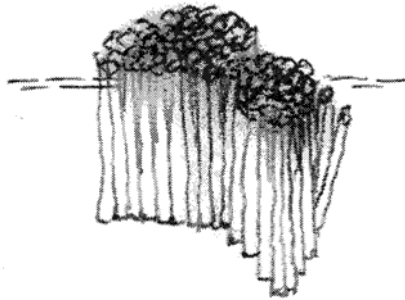
Figur 6 Superimposed snow-ice on a lake, milk-coloured.





### Behaviour of lake ice in a state of melting

Already at temperatures close to zero bonding between grains start to loosen up and creep deformation becomes exponentially larger than for colder ice. When air temperature or water temperature is higher than the melting point of the ice it is in a state of melting. Solar radiation is probably the most important parameter because it penetrates transparent ice. Columnar ice will soon become black as soon as the overlaying snow has melted away. As soon as meltwater is heated conductivity and steam erodes the remaining melting ice and it becomes permeable. It is commonly said “the ice is floating up” which is an important feature making ice fishing and other activities on the ice more pleasant. This type of ice is however not very reliable and will rapidly fall into pieces if the water temperature rises. When the ice turns black again it has almost no bearing capacity left even though it can be some 30 cm thick. This type of ice in melting condition is called *rotten ice*. It is during such spring periods one may enter the ice cover in the morning without any problems but the mid-day sunshine makes the ice dangerous.



Figur 7 Rotten ice in a state where the crystals are loosened up.

### **River ice**

(to be written later)

### **First-year sea ice**

#### Sea ice formation

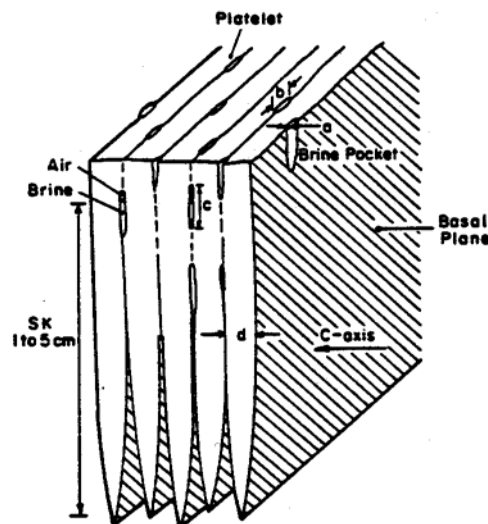
Standard seawater has a salinity of 35 psu (practical salinity units, 1 psu  $\approx$  1 part per thousand on mass basis). The phase-diagram shows linear decreasing freezing points for increasing concentrations of salt (NaCl) with typical  $-1.9^\circ\text{C}$  at 35 psu. Arctic Ocean surface water has a lower salinity (30-34 psu) and close to shore under ice river plumes decreases the surface salinities much more. Fresh water has a well-known density maximum at  $+4^\circ\text{C}$ . For sea water where the salinity is higher than 24.7 psu it becomes more dense as the temperature decreases. This leads to mixing of the whole water body during the time the surface water is cooled down. Therefore seawater freezes much later than fresh water with the same depths. In practice the very deep water is usually more dense than the surface water even when it is cooled down to its freezing point which means that about 10-20 m of the Arctic Oceans has to be cooled down to about  $-1.5^\circ\text{C}$  outside the continental shelf. After this first cooling period



*grease ice* is possible when small crystals of supercooled water start to float up to the surface. The name grease ice can be explained from the greasy and viscous appearance before the frazil (in calm conditions) has frozen together to *nilas* (swe. nyis). Due to wave action pancake ice or (swe. tallriksis) are formed with diameters that varies from 30 cm to several metres.

The ice becomes gradually thicker and more concentrated due to wave and wind action and the pack ice will eventually, dependent on air temperature, freeze together into *young ice*, 10-30 cm thick. In exceptionally calm and cold conditions nilas may keep unbroken until it reaches the thickness (> 30 cm) and strength to resist wind action. Then most of the young ice consists of columnar ice crystals similar to fresh-water columnar ice. One difference is of course that the ice crystals are frozen from salt water and salt is partly included into the ice crystals in a distributed pattern as *brine pockets*. Fortunately most of the salt is expelled during the freezing process (sea ice has a salinity of 5-10 psu) but this relatively low content is more than enough to effectively weakening the strength. When fresh ice crystals form in columnar ice, salt is expelled at the ice-water interface of growing platelets. This is how the brine initially becomes trapped and lined up between the platelets.

The vertical brine distribution in a sea ice cover keeps changing with time. There are many processes going on that are contributing to desalination: salt migration within the crystal, brine expulsion, gravity drainage and most important - flushing. Flushing becomes dominant during melt periods when brine pockets grow big and migrate to the nearest grain boundaries, where vertical brine channels form. The distance between these channels are thus about the same as the grain diameter but tubular channels may also penetrate the central parts of the grains at the bottom layer of an ice sheet.



Figur 8 Trapped salt as brine pockets between the platy ice structure. The distance  $d$  is usually less than 1 mm (Kovacs et al. 1987).

### Ridge formations

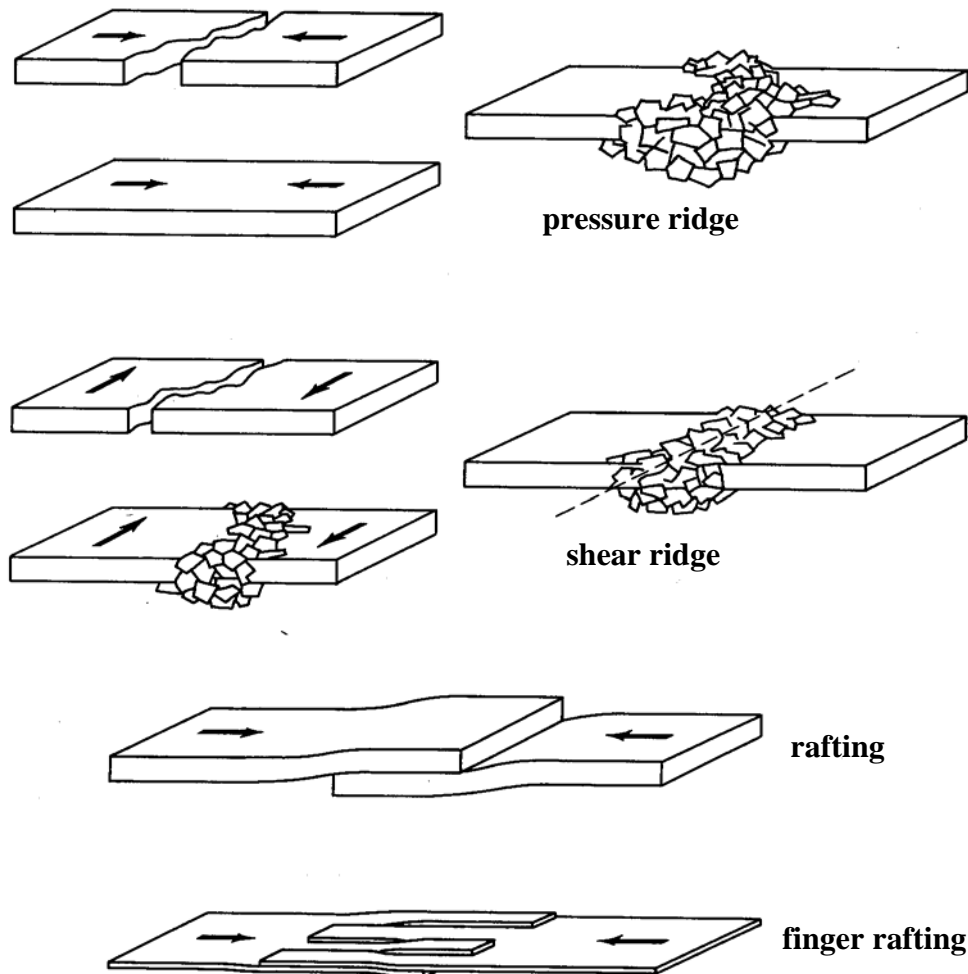
Due to a combination of thermal and dynamic processes a variety of ridge formations can be observed in first-year ice. Wind and wave action are the principal driving forces that create these interesting features which has such an importance for ice navigation as well as for other



travellers. The ridge formations can be sorted after ice concentration and breaking resistance according to:

1. Pack ice
2. Brash ice barriers
3. Finger rafts
4. Rafted ice
5. Shear ridges
6. Pressure ridges
7. Multiple rafts or hummocked ice
8. Grounded ridges

The resistance of the ridges depends mainly on size and the degree of consolidation, i.e. how much of the ridge that is frozen together. All ridges are thicker than the surrounding level ice which can be utilized as a method to detect them from satellite images or from automatic airborne ice thickness measurements. *Pressure ridges* are the single-most important ridge type for navigation. The formation process and the composition of pressure ridges has been studied by ice engineers all over the world. It is believed that thermal cracks or wave cracks is the weak line that breaks and initiates the ridge when the ice sheet is under lateral pressure. Once the process has started it will remain as a weak line for a long time. Ice floes are gradually broken (similar to upwards bending against a sloping structure) and will slide on top of each other. The final geometrical shape depends on the size and quality of the broken ice floes and the magnitude of the driving force. When the ridge grows to a certain size equilibrium is reached and new ridges are formed elsewhere. The ice portion observed over the water level is referred to as the *sail* and the portion under the water level is called the *keel*. The density of pressure ridges is usually high in the Gulf of Bothnia, which is a major problem for winter shipping. Due to prevailing southern and western winds the entrance to the harbours becomes clogged. A typical pressure ridge formed from 30 cm ice can be more than 10 m deep with a sail height of 2 m. During mild winters almost all ridges are unconsolidated which opens a possibility for icebreakers to navigate along the centre of the ridge.



Figur 9 Ridge formations numbered after breaking resistance (Sanderson 1988).

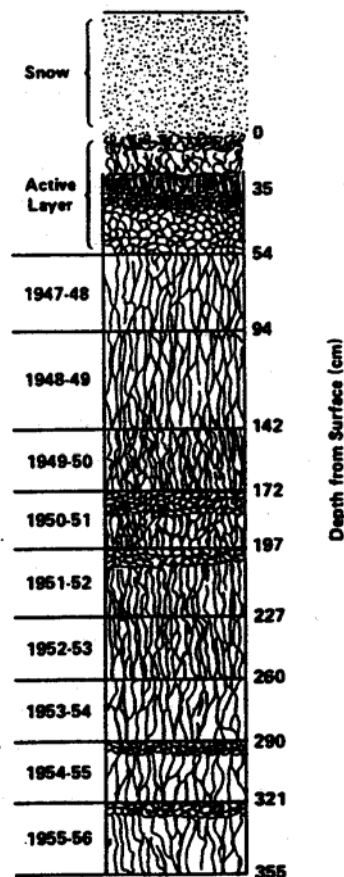
### Brackish ice

The water in the Baltic Sea has a salinity of 3-10 psu and Baltic sea-ice becomes more like fresh-water ice even if there are important differences. Salinity decreases as we go north with the lowest salinity in the Bothnian Bay. In the Luleå inner archipelago the ice is similar to lake ice with a land fast ice cover, but further out wind, waves and salinity (3 psu) makes the ice look like ordinary sea ice. Due to the shallow waters it is difficult to find water as warm as + 4°C, which implies that mixing of the water body is present during the ice formation. It should however be noted that the salt is not expelled in same amount as for ordinary sea ice, the salinity of brackish ice is about 1 psu in young ice. Pancake ice and snow-ice are commonly observed and these ice types are more saline than the columnar ice. In mild and snowy winters it happens that the ice cover becomes flooded due to the overburden. Freezing of the flooded snow involve concentration of brine in the intermediate water layer. Therefore the bonds between the layers become weak in mild temperatures. In the case of rafted ice the process is somewhat different because it is only a thin water layer between the floes that has to freeze.



### **Multi-year ice**

Multi-year ice can be defined as ice that has survived at least two summer seasons, which is only possible at the polar regions. Areas with extremely thick ice formed due to multiple rafting or due to other types of ridging like grounded and well consolidated ice ridges are the first candidates. As for glacier ice the snow cover plays an important role because of its ability to reflect solar radiation. During melt periods firn and meltwater are drained through the snow-pack until melt pools are formed. The *active layer* of multi-year ice (0.5 m in the arctic) undergoes drastically changes due to several thaw-freeze cycles. Brine channels develops and brine is transported downwards and will gradually be replaced with fresh-water from the melt pools. Under this active layer the old ice structure remains but brine channels are open summertime. New layers of columnar ice form wintertime to an even greater depth partly because of the low salinity of surface water. The total depth of multi-year floes in the polar pack ice is typical 2-6 m with an age of 5-10 years.



Figur 10 Vertical profile through a multi-year floe (Cherepanov 1957).

### **Glacier ice**

#### **Constitution of glaciers**

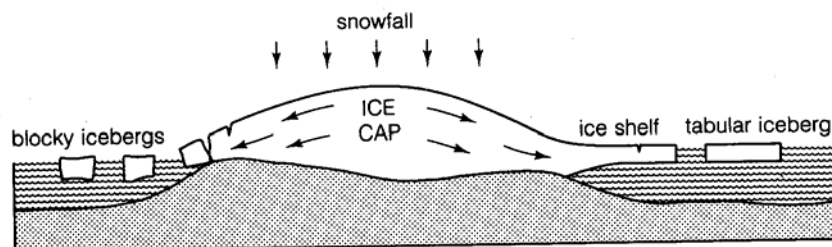
Glacial ice develops from snow, which compresses under its own weight until it becomes solid ice. Dependent on mean yearly temperature it needs to be different depths of snow before the snowpack turns to ice. Snow density at the surface is about  $400 \text{ kg/m}^3$  and the density increases gradually with depth at the same time as the snowpack becomes more like



granular snow-ice. The first 20-50 metres are usually referred to as *firn ice* because it is still softer and whiter than pure ice. Deeper down the glacier ice turns transparent and bluish or bluish-green if algae is embedded. It is important to separate between *cold glaciers* and *warm glaciers* mainly because of the different constitution and crack pattern. Antarctic glaciers are typical cold glaciers, i.g. they are developed under cold and dry conditions over millennia. The total thickness can be measured in kilometres and thus the crack widths and depths are large. Snow-bridges may hide cracked areas in such a way that even traverses with band-wagons become extremely risky. Warm glaciers in the Alps and in the Nordic mountains are younger with much smaller depths. Smaller vertical cracks and under-ice rivers are still severe threats to mountaineers because the surface is often covered with soft snow.

## Icebergs

Dependent on their origin we have two different types of iceberg: *blocky icebergs* and *tabular icebergs*. Blocky icebergs form by breaking off from sliding glaciers when they enter the ocean. The shape becomes blocky because the glacier is already pre-cracked when it reaches to the water due to steep slopes. Tabular icebergs are only possible in the regions where the glacier slides out in the ocean without breaking, forming the ice-shelf typical for Antarctica. Even the thickest ice-shelf has to break (due to wave action) when it becomes long enough. The length to thickness ratio varies a lot but is usually more than 10:1. Icebergs can also be classified by size where the smallest icebergs (< 1m) are called *growlers* and the very large icebergs (> 100 m) are called *ice continents*. A tabular iceberg that broke from the Ross Ice Shelf of Antarctica had an estimated length of 160 km.



Figur 11 Origin of icebergs (Sanderson 1988).

## Man-made ice

### Flooded ice

In order to make the ice surface smooth or if one wants to increase the growing speed of an ice cover it is common to use the technique of flooding in layers. Due to the fact that ice growth rate is faster for thin layers much efforts should be made to keep the layers as thin as practically possible. Still there are many differences between ice-making on natural ice outdoors compared with artificial freezing as in ice arenas:

- the ice surface is not perfectly horizontal
- the ice can freeze both upwards and downwards
- snow and wind may interfere with the freezing icelayer

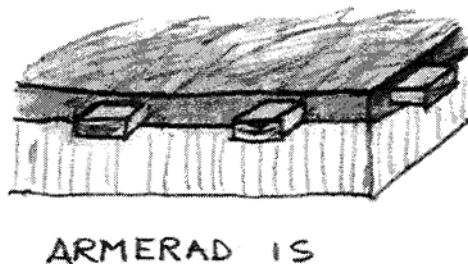


When many layers are made the temperature of the underlying ice becomes close to the melting point and it is impossible to avoid downwards freezing. This affects the bonding between the layers or the quality of the ice negatively because air bubbles concentrate in horizontal layers. Less air bubbles may form if a thin layer of heated water is smeared out with a rubber tool as done with modern ice making machines. This is not very practical if large volumes of ice are to be lifted by flooding. Instead it is recommended to keep a water thickness of a few centimetres and be sure that all the water that has been flooded stays where it is for the whole freezing time. All leakage (sideways and downward) must be avoided by making walls of ice or other materials and by closing the waterhole that has been used as water supply.

Flooded ice is always more sensitive to sun radiation than natural ice. It will soften and loses the bonds between the layers. This depends largely on the water quality of the flooded layers. Water pumps with high capacity tend to bring sediments that has to stay somewhere on the ice. Also the amount of air bubbles makes the ice more sensitive because they capture the energy of the incoming radiation. One method to prevent ice decay during hot spring periods is to protect the surface with snow or scraped ice from the surface if the ice is thick enough.

### Reinforced ice

During World War II the British army planed to construct a floating airport of reinforced ice (Gold 1989). In the tests that were done in Canada at that time it was found that wood was a perfect reinforcement material for ice. One large test ship was built of reinforced ice but the war ended before the construction of the full-scale hangar started. Ice was reinforced about the same way as concrete by putting bars with high tensile strength inside the moulded material. A natural ice sheet can be reinforced with a net of steel or wood frozen into the surface. In the experiment that took place at Luleå University of Technology 1979 we concluded that 1% relative cross-sectional area of wood was suitably to increase the bending capacity of ice beams. Another advantage is that reinforced ice becomes much more ductile than natural ice. Ice can also be reinforced by mixing fibres into the ice matrix. High strength and ductility can be achieved at least in laboratory experiments. In full scale there are several problems that have to be solved. Two difficult things are to prevent solar radiation to heat up the fibres and to find materials that are harmless for the environment.



Figur 12. Reinforced ice made with wooden bars.



## 2. Bearing capacity of ice

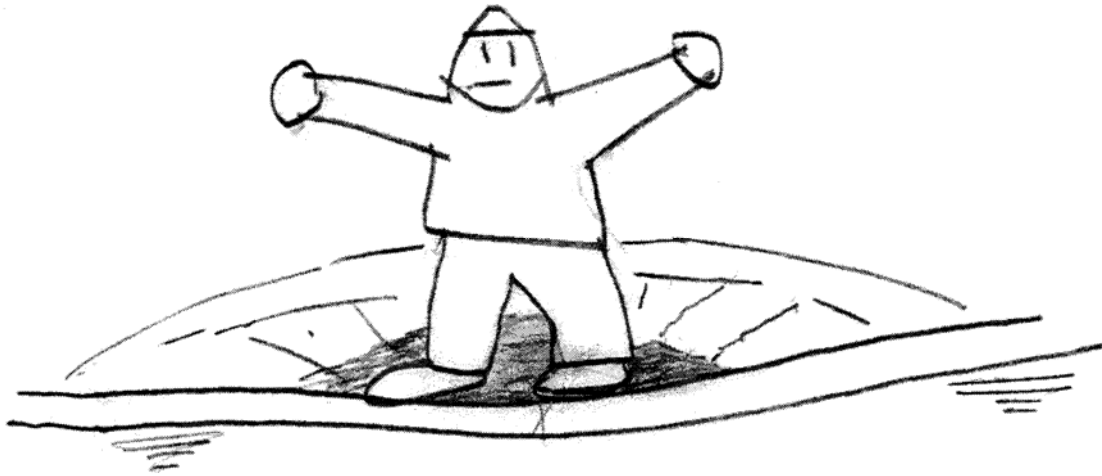
### ***Entering thin ice***

If we dare to enter really thin ice we will soon notice that it cracks for every step but if we are lucky the ice somehow still carries the load. During mild weather water may penetrate the cracks and it is possible to see the deformation disc. Extra weight from flooded water is bad news because the weight of the displaced water must be equal to our weights. It is then smart to distribute the load on a larger area and move so the stress on the cracked ice becomes smaller. As long as we have a thin watertight membrane on the water it can take about any distributed load but ice is a brittle material that easily cracks in bending. Fortunately these fine radial cracks propagate long distances without substantially decreasing the bearing capacity of the ice sheet. Cracks will however make the ice sheet more ductile for loads that are travelling the same trace as before.

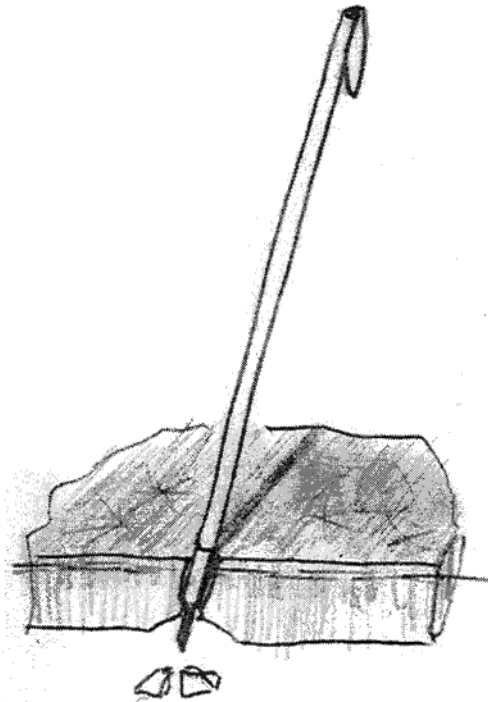
In a static loading test with rapidly increasing load level ice sheets tend to crack in a pattern that is similar to a bicycle wheel. First comes the radial cracks (visible below the ice) starting from the load contact and propagating radially to different lengths. It is usually up to 8 radial cracks but it depends on the geometric shape of the load contact. At a higher load level circumferential cracks appear at a distance from the load which depends on the ice thickness and the elastic modulus of the ice. At this stage the final failure is close especially on thin ice. Slow loading rate or constant load will cause more circumferential cracks closer to the load centre. The ultimate load is usually associated with a vertical punch through of the contact area. On ice with reduced shear strength and low elastic modulus (snow ice or melting ice) the punch trough may come before the circumferential cracks.

How do I know that the ice is strong enough to carry me? Ice thickness is of course the most important parameter for bearing capacity. In general 5 cm of uncracked fresh-water ice should be sufficient to carry a person less than 100 kg (SäkI 2000). It is however inconvenient to measure the thickness carefully every 10 metre if we want to skate on ice. Measuring thickness usually means that we have to cut a hole in the ice. It is also difficult to say exactly how thick or thin the ice has to be to carry a person. Bearing capacity depends on the ice type and quality and how this person behaves on the ice. As said before 30 cm of rotten lake ice may totally have lost its bearing capacity. A much better method is to sense the ice with a pole that has a sharp edge, an *ice-stick*. The weight of the ice-stick should be such as it is possible to penetrate weak ice without using much force. It is important to use the same type of ice-stick and to practice first on thin ice. You can for instance check thin ice on shallow water without risk other than getting wet feet. This device should not be mixed up with ice-prods that is a safety device that you hopefully never have to use but is important to carry in order to be able to get up from a hole in the ice. Remember that breaking through the ice on deep waters is as close to death you can get if you are inexperienced and alone.





Figur 13. Ultimate loading of a thin ice sheet.



Figur 14 The classic ice-stick for sensing the ice thickness and quality.

### ***Thin plate theory***

#### Calculation of deflection

Bearing capacity of ice can be studied by solving the governing differential equation for a thin elastic plate on elastic foundation. Hertz (1884) made an important contribution in his article: "Über das Gleichgewicht schwimmender, elastischer Platten" where he actually tested the validity of his solution by loading an ice cover. The solution showed correctly the total deflection disc from a point load on a infinite plate supported only by the water. In order to get reasonable stresses in the ice it is necessary to let the applied load be distributed on a



contact area greater than zero. By doing so solving the plate equation become more of a mathematical challenge but some important steps are given in the Appendix 1. In this Section only the final results are discussed. As for all homogeneous and isotropic materials the elastic modulus  $E$  and Poisson's ratio  $\nu$  is the only material properties needed for the ice cover. The uplift from the water is assumed proportional to the water pressure  $\rho_w g$ . From these constants it is convenient to define a new constant, the *characteristic length*  $L$ , that says something about how the ice sheet is deforming. The characteristic length of ice is derived from

$$L^4 = \frac{Eh^3}{12(1-\nu^2)\rho_w g} \quad (3)$$

In many engineering problem it is sufficient to use one set of constants where  $E = 4.5 \text{ GPa}$ ,  $\nu = 0.3$ ,  $\rho_w = 1000 \text{ kg/m}^3$  and  $g = 9.81 \text{ m/s}^2$ . Inserted into Eq.(3) the characteristic length can be approximated as  $L \approx 14 h^{3/4} \text{ [m]}$ . Notice that the ice thickness  $h$  also has to be in metres. An ice thickness of  $h = 0.1 \text{ m}$  it will result in  $L = 2.5 \text{ m}$ , this reflects on how far from the load we will see a deformed ice sheet. The central deflection  $w$  from a load  $P$  can be approximated as

$$w = \frac{P}{8\rho_w g L^2} \quad (4)$$

Using the same constants as before yield the simple relationship  $w \approx 0.065 Ph^{-3/2} \text{ [mm]}$ , where the load  $P$  is input in [kN] and  $h$ [m]. The deflection is directly proportional to the load. A load of 1 kN on 0.1 m ice results in a deflection of only 2 mm.

### Calculation of first crack load

Bending moment and stress was first used as a design criterion for ice in connection with the Trans-Siberian railway ice crossing over Bajkal guided by Bernstein (1929). Assuming linear strain distribution, maximum stress is  $\sigma = 6 M_0 / h^2$ , where  $M_0$  is the central moment proportional to the applied load. If the load is uniformly distributed over a circular contact area the maximum moment and thus the stress decreases with the relative load radius

$$\alpha = r / L \quad (5)$$

, where  $r$  is the load contact radius and  $L$  is the characteristic length, Eq.(3). The stress intensity function can be approximated with the function  $f(\alpha) = (0.6159 - \ln \alpha)(\alpha/2) + \pi\alpha^2/64 + \dots$ , and the crack load  $P_{cr}$ , defined from the crack criteria  $\sigma = \sigma_f$  becomes

$$P_{cr} = \frac{\pi\alpha}{3(1+\nu)f(\alpha)} \sigma_f h^2 \quad (6)$$

Where  $\sigma_f$  is the flexural strength of an ice sheet when the bottom is in tension.



This is usually determined from beam tests, where  $\sigma_f = 0.6$  MPa seems to be a typical value for river ice. With a load radius of  $r = 0.25$  m (a footprint) on 0.1 m ice result in  $\alpha = 0.1$  and a crack load

$$P_{cr} = 0.55 \sigma_f h^2. \quad (7)$$

Using these results gives a typical crack load of 0.0033 MN or 3.3 kN for a 0.1 m thick river ice sheet and a crack load of 0.825 kN on 0.05 m ice. It seems thus likely that 5 cm ice will crack sometimes when a skater or a heavy person enters the 5 cm ice.

### Semi-empirical formula for break-through load

Fortunately first crack appearance doesn't always mean that it is close to break-through. Experiments have shown that substantially higher loads are possible on an infinite floating plate. Panfilov (1960) assumed that there is a linear relationship between failure load and the relative load radius that is more or less parallel with the first-crack function, Eq.(6), at least when  $\alpha$  is larger than 0.1. For more concentrated loads the difference is more pronounced because the maximum stress goes to infinity. The failure load or the ultimate load according to Panfilov (1960) is

$$P_u = 1.25(1+1.68\alpha) \sigma_f h^2 \quad (8)$$

A relative load radius of 0.1 results in

$$P_u = 1.46 \sigma_f h^2 \quad (9)$$

Using the same example as before for 5 cm ice we will get an ultimate load of 2.2 kN which is twice the weight of most persons. Even if the load is extremely concentrated as from skates we can expect an ultimate load of  $1.25 \sigma_f h^2$  or 1.9 kN. It is however important that the ice quality (shear strength) is good enough to avoid a punch through failure.



### 3. Ice loads on structures

#### *Ice conditions and structures*

##### Extreme ice conditions

The term “extreme ice condition” aims in this section on some kind of worst-case scenario which also depends on the structure we have in mind. Ice conditions all over the world have been mapped systematically for more than 100 years but in remote areas the old data are sparse and unreliable and dependent on heroic expeditions. Today’s global satellite information systems are extremely useful for ice mapping and together with sub-marine measurements a fantastic database is available about the Arctic ice conditions. Arctic ice and also climate is governed by two major movement systems: the *Transpolar Drift* and the *Beaufort Gyre*. The transpolar drift runs across the Arctic Ocean from Siberia to the Fram Strait between Greenland and Svalbard (Nansen, 1893-1896). Air temperatures and solar radiation and thus the ice thickness of multi-year ice is a only partly a function of altitude. The ice extent and thickness is also strongly affected by the warm Gulf Stream which is the reason the Norwegian sea is kept open all year around. Icebergs are most frequently seen around the coast of Greenland but some icebergs have been sighted on odd places like offshore Ireland, 1907 and at the Norwegian coast 1929.

The Baltic Sea is probably one of the areas where we have the most detailed ice statistics in the world. The shallow brackish water cools down earlier than ordinary oceans with similar climate, which creates relatively large coverage of strong first-year ice. Statistics from SMHI (Sveriges Meteorologiska och Hydrologiska Institut, Norrköping) is available as yearly report of the ice season and as an Ice Atlas. There has been a sequence of relatively mild winters during the last decade but such fluctuations have been observed earlier. 1985 was a severe winter with about 1 m level ice in the Gulf of Bothnia. This might have been the reason why lighthouse Björnklacken, close to Luleå harbour this winter was overloaded and tilted by an unexpected ice movement. Further south the ice conditions become gradually less severe, but some year the whole Baltic Sea may have been covered with ice, strong enough to travel on.

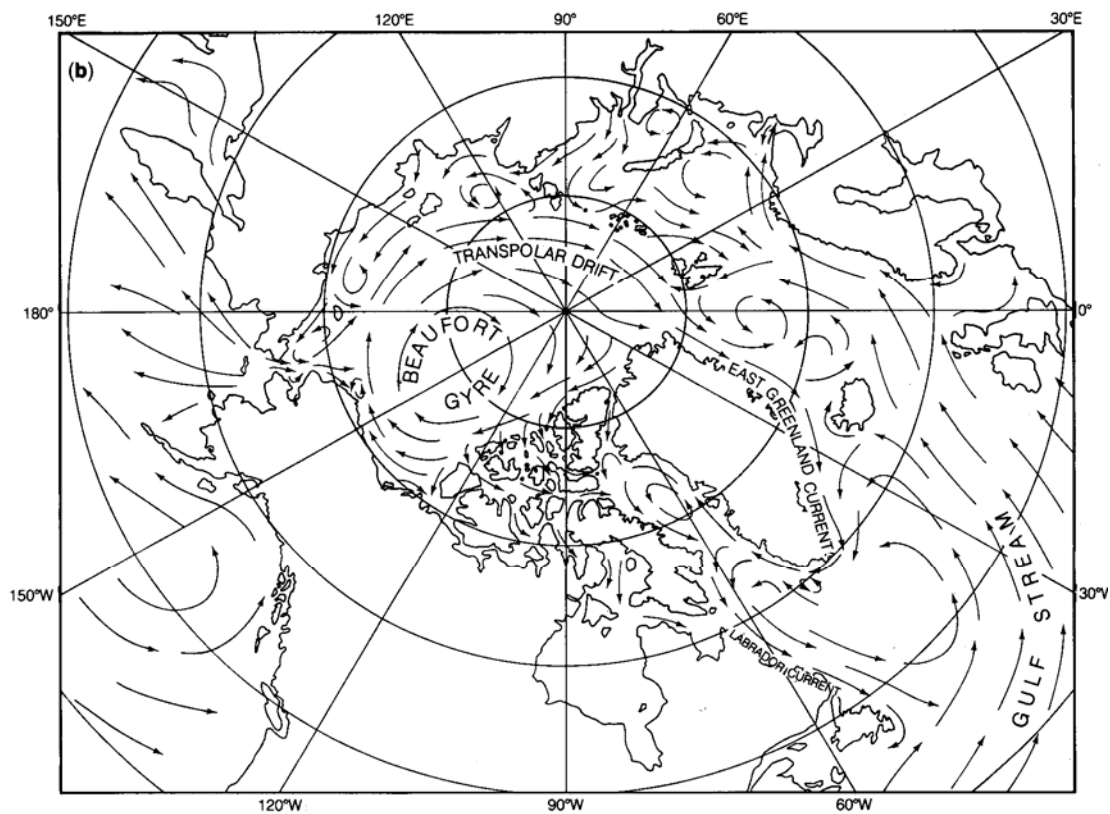
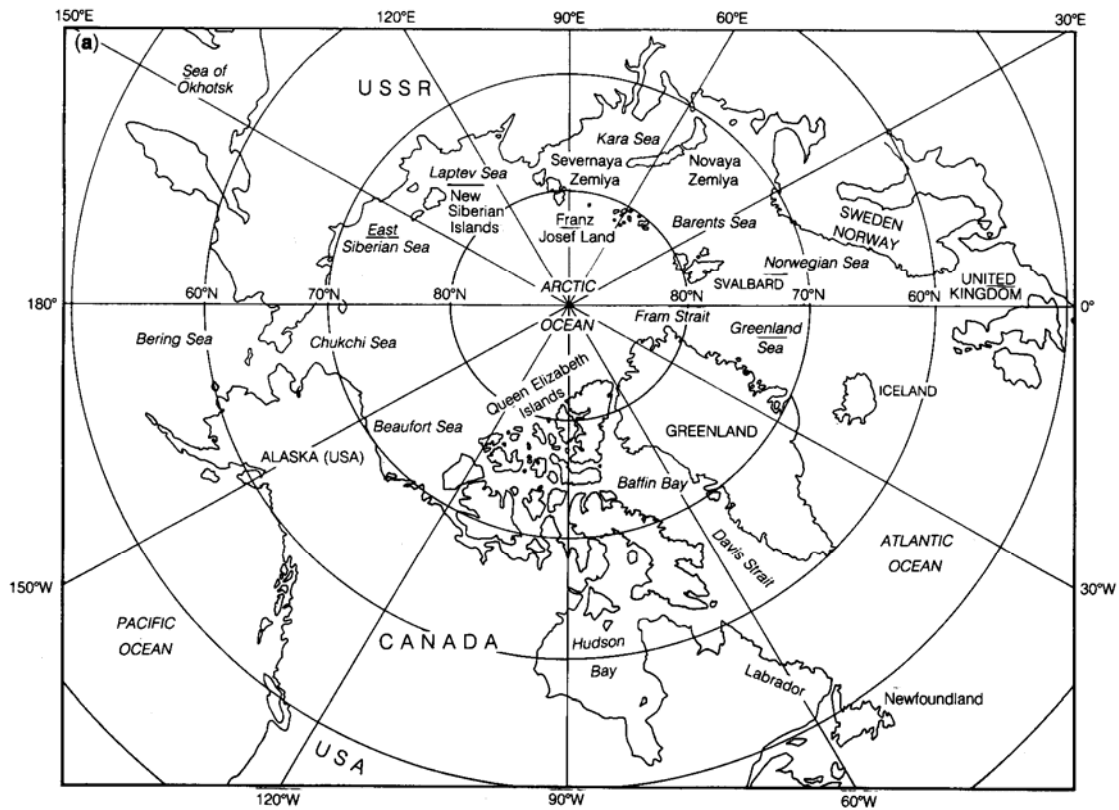
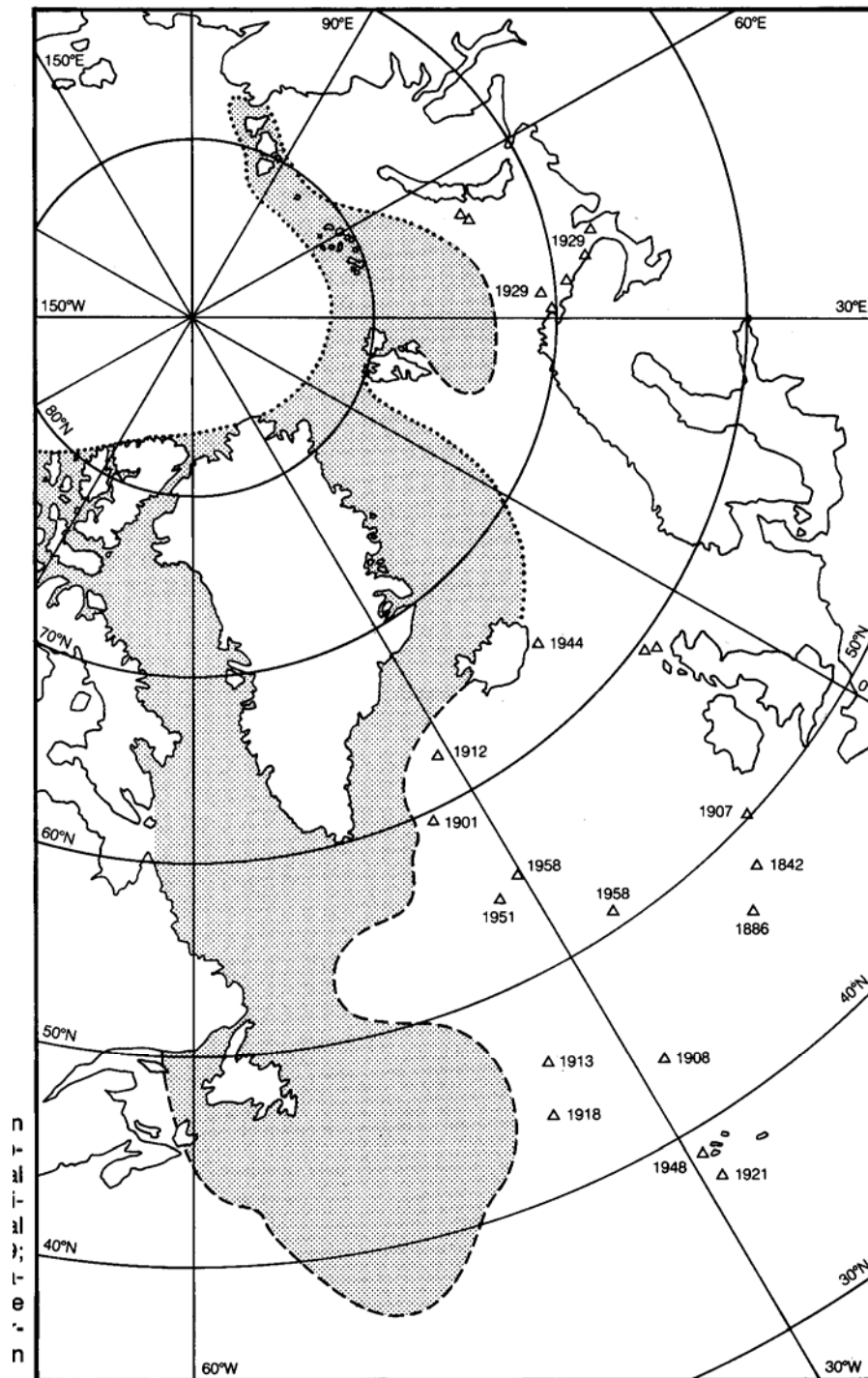


Fig. 3.2 (b)

Figure 15 Map over the northern hemisphere (a) and the Arctic streams (b) (Sanderson 1988).



Figur 16 Occurrence of icebergs (shaded area) and odd locations plotted as triangles (Sanderson 1988).

### Typical structures in ice

The most common structures sustained to ice are probably bridge piers, harbours and reservoir dams. Later on some lighthouses for navigation purpose in the north were built



offshore. Today the applicability of offshore-based wind-generators has been discussed even for coastal areas outside Luleå.

### Ice failure modes

Creep  
 Creep buckling  
 Elastic buckling  
 Creep crushing  
 Continuous crushing  
 Horizontal splitting  
 Radial cracking  
 Bending failure  
 Non-simultaneous failures

### ***Static load from an ice sheet***

#### Thermal ice pressure

Ice covers may expand several meters during a winter season. This one-way thermal expansion goes on like this:

1. wet cracks form when a confined ice cover cools down
2. cementation of cracks during cold periods
3. the ice sheet expands at the surface proportional to temperature change
4. ice expansion causes pressure on all fixed supports (shore-line, islands, structures)
5. ice moves towards weak zones (open water, channels, thin ice, wide cracks)

It is possible to estimate the maximum thermal pressure by looking at the thermal potential. Cold areas with little snow are most likely to face these problems. Thermal ice load  $F_{\text{thermal}}$  is also dependent on more parameters (ice thickness) but as a first approximation the formula

$$F_{\text{thermal}} = k_T (-T_0)^{1.5} D \quad (10)$$

can be used, where  $k_T$  is an empirical constant,  $T_0$  is the initial ice surface temperature and  $D$  is the structure width. Fransson (1988) suggests  $k_T = 2 \text{ kN, m}^{-1}, \text{ }^\circ\text{C}^{-1.5}$  based on his field measurements in Lule river and Skellefte river. In northern Sweden it is reasonable to choose  $T_0 = -20 \text{ }^\circ\text{C}$ , which results in  $F_{\text{thermal}} = 180 D \text{ [kN]}$ .

#### Onset of ice movement

Static ice loads on interacting structures are assumed to reach a maximum just before the ice is about to move. This is not always the case because stress release may come earlier due to radial cracks or creep buckling. If the contact between ice and structure is smooth the stress distribution becomes rather uniform after a certain loading time due to viscous creep and creep crushing. The highest stress level is a function of uniaxial compressive strength  $\sigma_c$  and the degree of confinement  $k_c$  of the ice in front of the structure. From plasticity analysis it can be proved that this confinement coefficient  $k_c$  is between 1 and 3. These assumptions make it easy to calculate the static ice load as



$$F_{\text{static}} = k_c \sigma_c Dh \quad (11)$$

, where  $k_c = 1 - 3$  and  $\sigma_c$  is the average uniaxial compressive strength of the total ice thickness. The experimental procedure for testing ice strength must be designed to emulate ductile ice behaviour or creep crushing for mild or deteriorated ice. The load should be applied horizontally (perpendicular to the growing direction). For fresh-water ice the strain rate should be less than  $10^{-3}$  in order to produce yielding. Finally it is important to perform the tests as quickly as possible after sampling in a controlled temperature and humidity to maintain the original ice properties. It might be so that it is better to test larger volumes of ice than small samples if the crystals are big. Still it is not believed that there is any strong size effect on the compressive strength. Therefore also the static ice pressure is assumed independent on size of the pressure area if the confinement is constant.

### ***Dynamic load from a moving ice sheet***

#### **Ice crushing**

If the driving force is sufficient large ice floes are crushed against vertical structures when they are moving. The driving force in the Gulf of Bothnia is wind and the drifting velocity is at the most 3% of the wind velocity, usually less than 0.6 m/s. At other ice covered oceans current and tidal variations also contribute to the ice drift. As said before ice is an extremely brittle material even close to the melting point and thus brittle ice crushing is a most common failure mode. Fracture mechanical behaviour implies that the process is highly dynamic causing substantial vibrations to almost any structure in interaction. These vibrations may lead to ice-induced resonant oscillations on flexible structures. Such oscillations are only limited by the damping in the system and may cause a total collapse of otherwise imposing structures.

In order to understand the nature of ice crushing it is necessary to study the contact phenomena going on in the ice. Ice crushing experiments with transparent indentors have shown that *high pressure zones* develops at the centre of the contact area (Joensu & Riska, 1988; Fransson et al. 1990). The maximum pressure in these zones are probably limited only by pressure melting (about 100 MPa for ice with a temperature of  $-10$  °C). Average pressure during ice crushing is typical less than 1/10 of these high pressures. The impact of a strong pressure gradient on the contact area seems to be the explanation why continuous ice crushing changes to intermittent crushing with creep deformation when the process is too slow to maintain the high pressure. Even though the process is still poorly understood, Fransson & Stehn (1993) proposed that the load from ice edge crushing is limited due to horizontal cleavage cracks in front of a vertical structure. The failure load  $F_{\text{split}}$  is given by

$$F_{\text{split}} = k_w K_{\text{ic}} Dh^{0.5} \quad (12)$$

,where  $k_w$  is a function of the relative width ( $w$ ) of the high pressure band,  $K_{\text{ic}}$  is the fracture toughness of ice,  $D$  is the width of the indenter and  $h$  is the ice thickness. It should be noted that the formula predicts that the effective ice pressure  $F/Dh$  is proportional to  $h^{-0.5}$ . It has been known for decades that measured average ice pressure is decreasing with the pressure area  $Dh$  but it is still discussed among ice engineers exactly how this pressure decrease can be





explained in full scale. In nature ice pressure can be limited by many other failure modes (bending, splitting or collapse of weak ice in inhomogeneous ice). Thin ice ( $h < 0.1$  m) seems to depend on its crystal structure, it behaves differently in downscaled laboratory tests. Also the load during ice edge crushing at very high speed with stiff indentors seems to be limited by small-scale axial splitting but it is unclear how the load is related to the ice thickness. Crushing load is often characterised as a saw-tooth curve where the force drops faster than it builds up. For wider structures this high frequency variation is overlaid a certain quasi-static load level usually referred to as *ice extrusion* pressure. The structure is then always in contact with the ice edge.

### Upwards bending failure

For a wide structure with an inclined foundation a moving ice sheet will slide up until it breaks in upwards bending. The total horizontal load on a sloping structure comprises mainly two parts: *breaking load* and *ride-up load*. The relationship between horizontal and vertical forces ( $F_H$  and  $F_V$ ) on a sloping structure is given by

$$F_H = N \sin \alpha + \mu N \cos \alpha \quad (13a)$$

$$F_V = N \cos \alpha - \mu N \sin \alpha \quad (13b)$$

, where  $\alpha$  is the slope ( $\alpha = 90^\circ$  is a vertical structure),  $\mu$  is the friction coefficient and  $N$  is the normal force (normal to the foundation wall), see also Figure 17. Bending strength for ice  $\sigma_f$  was given in the bearing capacity chapter and it was shown that bending failure occurs at small strains and stresses. When the ice is lifted up at the edge it will crack at a certain distance (proportional to the characteristic length) from the structure. Seen as a 2-dimensional process (as for a long structure) it is relatively easy to show that the breaking load is proportional to  $h^{5/4}$ , see Eq. (14) below and Appendix 2. The average ice pressure will thus increase with ice thickness. Thick ice is always possible in deep waters as long as pressure ridges or other thick ice features are present. Then the ice failure mode will change to ice crushing on some distance from the structure. In shallow water and especially when the ice movement direction is constant (as for rivers) it is usually economical to choose an inclined foundation. The most important advantage is that bending usually occurs without risk of stationary ice-induced vibrations close to the resonant frequency of a well-designed structure. The breaking load  $F_{break}$  is written

$$F_{break} = C_1 \frac{\sigma_f}{L_1} D h^2 \quad (14)$$

, where  $C_1$  is a function of friction and slope and  $L_1$  is the characteristic length of a floating ice beam. The ride-up load is a function of the height  $Z$  that the weight of the ice floes has to be pushed up the wall of the foundation. This is a process that depends on time and eventually friction may increase drastically if it is cold enough for the floes, snow and water to refreeze on the structure. The ride-up load  $F_{ride-up}$  can be expressed as

$$F_{ride-up} = C_2 Z \rho_{ice} g D h \quad (15)$$

, where  $C_2$  is another function of friction and slope, different from  $C_1$ . The effect of friction is quite significant which can be illustrated by the following example with a structure inclined to  $\alpha = 45^\circ$  sustained to a uniform ice. Lets (a) assume  $\mu = 0.1$  and (b)  $\mu = 0.5$  and that the other



parameters remain constant. The effect of friction for a 2-D structure is given in Table 1 by the variation of constants. In this case the loads are more than doubled if the friction increases from 0.1 to 0.5. The friction effect becomes exponentially higher for steeper structures which leads to the conclusion that this simplified analysis should be avoided when  $\alpha > 50^\circ$ .

Table 1. Effect of friction on a sloping structure 45 deg.

$\mu$	$C_1$	$C_2$
0.1	0.8	1.7
0.5	2.0	4.3

For a narrow conical structure the breaking pattern is somewhat different from the 2-D case. First of all the ice sheet will crack in the radial direction out from the structure and the ice wedges are then bended upwards to failure at a fairly constant distance perpendicular to the structure. If the ice sheet is thick bending of circular cracks becomes harder after that the initial crack has been developed because a geometrical mismatch along the crack. Another difference is that the breaking load is more dominant compared with the ride-up load and thus the ice strength becomes more important. All together analytical solutions are more difficult to perform which leads to empirical formulas based on physical model experiences with different conical structures and ice conditions, etc. An early example of formula (Afanasev et al.1971) for narrow upwards-breaking cones was based on small-scale experiments with 3.5 cm ice. The breaking load  $F_{break}$  was calculated as

$$F_{break} = \frac{S_x \tan \alpha}{1.93L} \sigma_f h^2 \quad (16)$$

, where  $S_x$  is the length of the circumferential crack given as

$$S_x = 1.76 (D/2 + \pi L/4) \quad (17)$$

, where  $D$  is the cone diameter at the water level and  $L$  is the characteristic length of an floating ice sheet defined by Eq.(3). Several experimental studies have shown that the horizontal load for a narrow conical structure can be expected to depend on  $h^2$  and also Eq. (16) tends to do so when  $D \ll L$ . Ice pressure will then increase with  $h$  until other failure modes than upwards bending limit the pressure level on the structure.

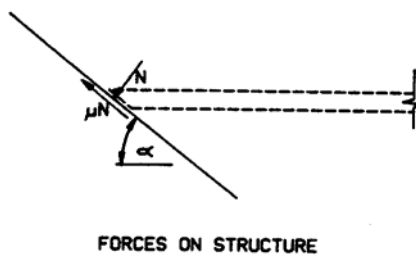
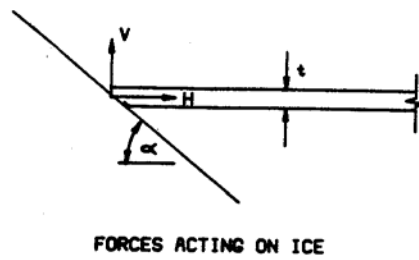


Figure 6. Initial interaction between ice and sloping structure.

Figure 17 Breaking forces on a sloping structure (Croasdale 1980, p45).

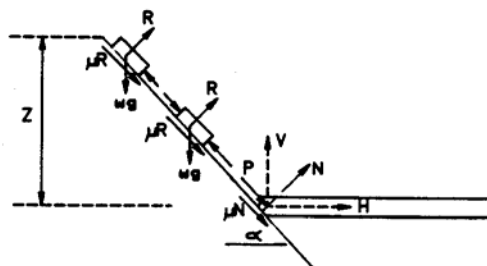
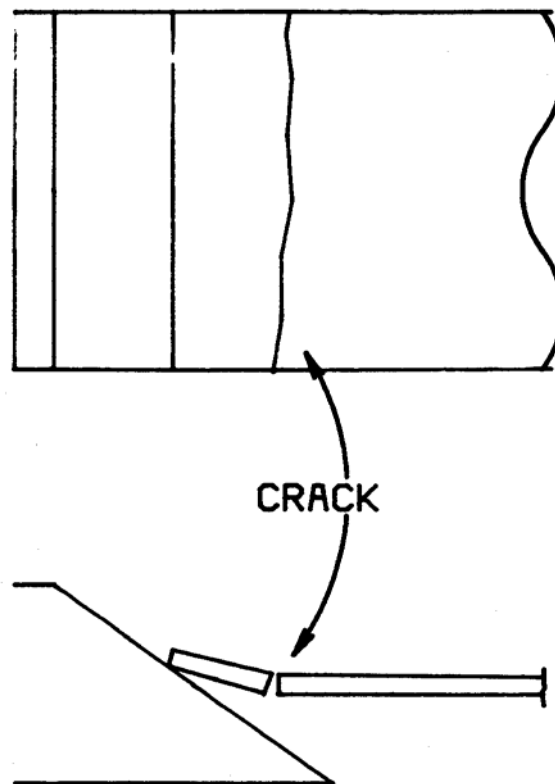
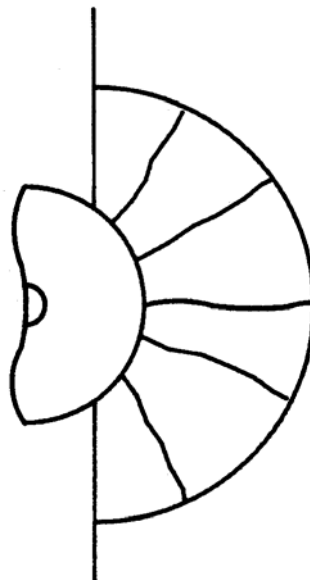


Figure 7. General interaction between ice and sloping structure.

Figure 18 Ride-up forces on a sloping structure (Croasdale 1980, p45).



Figur 19 Typical 2-dimensional upwards-bending failure. The crack length is equal to the structure width.



Figur 20 Typical 3-dimensional upwards-bending failure. The crack length is greater than the structure width.



### Radial splitting

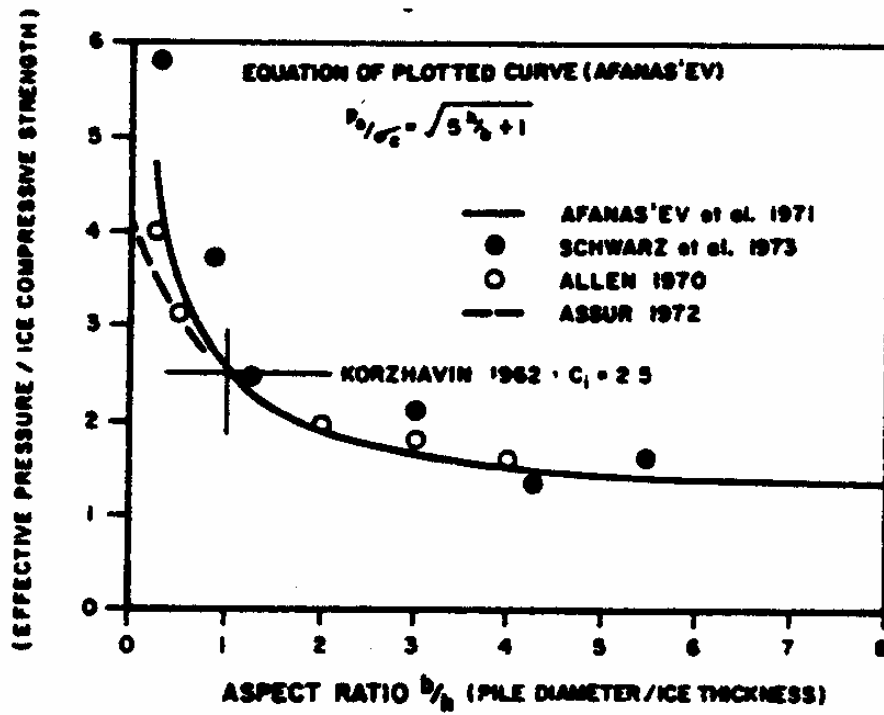
When an ice floe hits a narrow structure cracks tend to propagate radially. The distance these cracks may travel can be several kilometres if the ice is cold and brittle. Therefore smaller ice floes will be split into two halves or into more segments when the load becomes high enough. It is difficult to estimate the load associated with radial splits because the tensile stress depends on brittleness, confinement and size of interacting ice floes. In a river during spring break-up radial splitting usually limits the maximum ice load on the bridge pier if the width of the pier is made small enough. The ice strength also has to be low in most rivers before the ice is able to move. One might also think that splitting will be promoted if the foundation is designed like a sharp arrow. With the assumption that radial cracks are propagating due to stress concentrations at the contact zone it seems likely that the splitting force is proportional to  $(D_{\text{floe}})^{0.5}$ , where  $D_{\text{floe}}$  is a certain width of the interacting ice floe perpendicular to the ice movement.

### Non-simultaneous failures

For wide structures it is obvious that the pressure peaks measured on different small contact widths will not always occur at the same time. From this trivial observation it is a long way to the conclusion that it will never happen. For a stiff structure the ice pressure is only a function of the ice failure process. But as the structure deforms or moves all parts on the total width can be locked-in, i.e. experience resonant oscillations. If so, all peak pressures are likely to be in phase with each other. The failure process itself is also far from random, even though they appear to be. Yielding at the corners of a narrow structure is a more confined process than yielding at the front of a wide structure. This can be partly considered with Afanasev's amendment to Eq.(11) where the confinement  $k_c$  varies with the aspect ratio  $D/h$  according to

$$k_c = (5h/D + 1)^{0.5} \quad (18)$$

One must remember that the confinement is only this big in a situation where the structure is surrounded by an uncracked ice sheet as in the static loading case. When  $1 < D/h < \infty$ , this function is between 2.5 and 1. A similar function of aspect ratio can be expected in connection with continuous crushing on vertical structures due to the observed higher maximum pressure levels at the corners of a flat indenter (Fransson & Nyström 1994). This effect was pronounced on a distance equal to the ice thickness from each corner. Narrow cylindrical structures are affected by the same phenomena but the pressure distribution over the structure width is different. Extrusion of crushed material prevents the pressure to go below a certain minimum level independent on structure width. Very little is known about how this quasi-static extrusion level varies with ice thickness and velocity.



Figur 21 Effective pressure vs. aspect ratio  $D/h$  (Määttänen 1980, p116).



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